

Small Loop and Analysis Calculation For the Small HF Loop

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Note that the loop is in the XY plane in this analysis!

This Mathcad-based analysis is the basis of the small loop compliance distances published in FCC OET65 and "RF Exposure and You," ARRL, Newington CT, 1998. This analysis uses current and electric charges for small loops calculated from:

- J. E. Storer, "Impedance of thin wire loop antennas," NRL Rep. 212, Cambridge MA, May 1, 1955.
- K. Siwiak, "Loop Antennas," in John G. Proakis (Ed.), **Wiley Encyclopedia of Telecommunications**, New York, NY: John Wiley & Sons, 2002, pp 1290-1299.
- Q. Balzano, K. Siwiak. "Radiation of Annular Antennas," *Correlations*, Motorola Engineering Bulletin, Motorola Inc., Schaumburg, IL, USA, Volume VI, No. 2, Winter 1987.
- Q. Balzano, K. Siwiak. "The Near Field of Annular Antennas," *IEEE Transactions on Vehicular Technology*, Vol. VT-36, No. 4, pp. 173-183, November 1987.
- J. H. Dunlavy Jr., "Wide range tunable transmitting loop," U. S. Patent 3,588,905, 28 June, 1971.

Fields equations are from:

- Ronold W. P. King, Charles W. Harrison, Jr., *Antennas and Waves: A Modern Approach*, MIT Press, Cambridge, MA, 1969.

The loop is circular with radius b meters and wire radius a meters and is **oriented in the xy plane** with a feed gap on the x -axis ($\phi=0$ deg). The wire cross section is measured by angle ψ with $\psi=0$ deg aligned with the x -axis. Detail of the current along the cross section of the wire is shown and can be included for results that are valid to within a wire radius of the antenna.

The power supplied to the loop can be adjusted by a factor of $F=1.6^2$ to account for a worst case ground reflection, but the ground reflection is NOT otherwise explicitly calculated.

For specific cases: enter power P watts, loop wire radius a , loop radius b meters, frequency f MHz, and loop conductivity σ mho/m.

In this specific case results are for a 0.90678 m diameter loop of 0.320 inch diameter aluminum wire at 7-30 MHz. The loop is constructed from RGC-213 coax; outer conductor is aluminum foil over copper braid. Inner copper conductor not included. This is close to the dimensions of the AlexLoop by Alex Geimberg, PY1AHD.

Loop Radiation Fields

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Loop is in the xy plane, current is non-constant, $a < b < \text{wavelength}$

FCC ground field factor: 1.6

$$F := 1.6^2 \quad F := 1$$

dimensions in meters

Power supplied, W

$$P_w := 5$$

$$P := P_w \cdot F$$

$$P = 5$$

Ground factor accounted for
by increasing power to loop;
all fields, currents, voltages
then use P, not P_w , in free
space.

Loop radius, m

$$b := 35.7 \cdot 0.5 \cdot 0.0254$$

$$b = 0.45339$$

Wire radius, m

$$a := 0.320 \cdot 0.5 \cdot 0.0254$$

Freq, MHz:

$$f := 14.1$$

$$a_{\text{inner}} := \frac{a}{2.84} \cdot 0$$

conductivity, mho/m: $\sigma := 34 \cdot 10^6$

$$\sigma_{\text{in}} := 58 \cdot 10^6$$

Cu is 58×10^6 , Aluminum is 34×10^6 mho/m

Added resistance:

$$R_a := 0.0$$

Tin is 8.7×10^6 ; solders are $\sim 8 \times 10^6$ Alloy AIC12200 is 80%Cu = 46×10^6

The estimated loop matching circuit Qmatch (unloaded), not including loop resistivity is:

Qmatch includes capacitor Q:

$$Q_{\text{match}} := 2400$$

(use 10^9 for no matching network losses)

$$c := 299792456$$

$$\mu := 4 \cdot \pi \cdot 10^{-7}$$

$$\epsilon := \frac{1}{\mu \cdot c^2}$$

$$\lambda := \frac{c}{f \cdot 10^6}$$

$$\Omega := 2 \cdot \ln\left(\frac{2 \cdot \pi \cdot b}{a}\right)$$

$$\omega := 2 \cdot \pi \cdot f \cdot 10^6 \quad k := \omega \cdot \sqrt{\mu \cdot \epsilon}$$

$$\eta := c \cdot \mu$$

$$\delta := \sqrt{\frac{2}{\omega \cdot \mu \cdot \sigma}}$$

$$Z_{\text{loss}} := \left[\frac{2 \cdot \pi \cdot (a \cdot \delta \cdot \sigma)}{2 \cdot \pi \cdot b} + \frac{2 \cdot \pi \cdot (a_{\text{inner}} \cdot \delta \cdot \sigma_{\text{in}})}{2 \cdot \pi \cdot b} \right]^{-1} \cdot (1 + j)$$

inner and outer
coax conductors
in parallel

Computed constants:

$$R_{\text{loss}} := \text{Re}(Z_{\text{loss}})$$

loop conductor resistance

$$\frac{a}{\delta} = 176.8$$

$$k = 0.296$$

$$R_{\text{loss}} = 0.143$$

$$a \cdot 1000 = 4.064$$

mm

$$\delta$$

$$k \cdot b = 0.134$$

$$\Omega = 13.105$$

$$b = 0.45339$$

m

(Z, including higher order

terms for, $(kb) < 0.4$.)

M12 HF model from LOOP-M12-coupling-to-smaller-loop-ZX-plane.mcd

$$M12 := 5.73045 \cdot 10^{-8} - j \cdot 2.06145 \cdot 10^{-11}$$

$$Z(k, b, a) := \eta \cdot \frac{\pi}{6} \cdot (k \cdot b)^4 \cdot \left[1 + 8 \cdot (k \cdot b)^2 \right] \cdot \left[1 - \left(\frac{a}{b} \right)^2 \right] + j \cdot \omega \cdot \left[\mu \cdot b \cdot \left[\ln\left(8 \cdot \frac{b}{a} \right) - 2 + \frac{2}{3} \cdot (k \cdot b)^2 \right] + M12 \right] \cdot \left[1 + 2 \cdot (k \cdot b)^2 \right]$$

This impedance is for ONLY

the loop, low freq inductance: $Z1(k, b, a) := \eta \cdot \frac{\pi}{6} \cdot (k \cdot b)^4 + j \cdot \eta \cdot k \cdot b \cdot \left(\ln\left(8 \cdot \frac{b}{a} \right) - 2 \right)$ $Z1(k, b, a) = 0.064 + 241.981i$

$$Q_{\text{rad}} := 0.5 \cdot \frac{\text{Im}(Z(k, b, a))}{\text{Re}(Z(k, b, a))}$$

loaded radiation Q

$$Q_{\text{rad}} = 1.72 \times 10^3$$

loop impedance (no loss):

$$Z(k, b, a) = 0.07458 + 256.55361i$$

$$\eta \cdot \frac{\pi}{6} \cdot (k \cdot b)^4 = 0.06357$$

loop impedance including wire
skin depth and added loss:

$$Z_{\text{loop}} := Z(k, b, a) + Z_{\text{loss}} + R_a$$

$$Z_{\text{loop}} = 0.21733 + 256.69636i$$

$$\frac{\text{Im}(Z_{\text{loop}})}{\omega} = 2.897 \times 10^{-6}$$

The total loaded (transmitter connected) QL includes Qrad, loop resistive loss Q, and the Qmatch:

$$QL := \frac{0.5}{\frac{1}{Q_{\text{match}}} + \frac{\text{Re}(Z_{\text{loop}})}{\text{Im}(Z_{\text{loop}})}}$$

$$QL = 395.78706$$

$$R_r := \text{Re}(Z(k, b, a))$$

$$Q_{\text{match}} = 2.4 \times 10^3$$

$$R_r = 0.075 \quad \text{radiation resistance}$$

$$Q_{\text{rad}} = 1.72 \times 10^3$$

$$R_{\text{loss}} = 0.143 \quad \text{conductivity loss}$$

$$I_o := \sqrt{\frac{P}{R_r} \cdot \frac{QL}{Q_{\text{rad}}}}$$

$$I_o = 3.928 \quad \text{A, rms}$$

$$\text{eff} := \frac{I_o^2 \cdot R_r}{P}$$

$$\text{eff} \cdot 100 = 23.012 \quad \%$$

$$\frac{QL}{Q_{\text{rad}}} \cdot 100 = 23.012 \quad \%$$

$$C_{\text{resonance}} := \frac{1}{\omega \cdot \text{Im}(Z_{\text{loop}})}$$

$$10 \cdot \log(\text{eff}) = -6.381$$

$$\frac{f \cdot 1000}{QL} = 35.625 \quad \text{kHz BW}$$

$$C_{\text{resonance}} \cdot 10^{12} = 43.973 \quad \text{pF}$$

$$V_{\text{cap}} := \sqrt{2 \cdot |\text{Im}(Z_{\text{loop}})| \cdot QL \cdot P_w}$$

$$\frac{V_{\text{cap}}}{\sqrt{1}} = 1007.954 \quad \text{Peak volts}$$

Current Density, wire; radius a, loop radius b:
 ϕ = loop circumference, 0 = feed gap
 ψ = wire circumference, 0 = outside of loop
 Loop is in x-y plane

$$Y(a, b) := -\left(\frac{2 \cdot a}{10 \cdot a + b}\right)^{0.75}$$

$$J(\phi, \psi) := \frac{I_o}{2 \cdot \pi \cdot a} \cdot [1 - 2 \cdot \cos(\phi) \cdot (k \cdot b)^2] \cdot (1 + Y(a, b) \cdot \cos(\psi))$$

The loop current variation around the wire with a feed gap on the +x axis is:

$$I(\phi) := I_o \cdot [1 - 2 \cdot \cos(\phi) \cdot (k \cdot b)^2] \quad J(\phi) := \frac{I_o}{2 \cdot \pi \cdot a} \cdot [1 - 2 \cdot \cos(\phi) \cdot (k \cdot b)^2]$$

$$q(\phi) := \frac{1}{-j \cdot \omega \cdot \rho} \cdot \frac{d}{d\phi} I(\phi)$$

$$q(\phi) := \frac{I_o \cdot (k \cdot b)^2}{-j \cdot \omega \cdot b} \cdot 2 \cdot \sin(\phi)$$

$$E_s(\phi) := \frac{q(\phi)}{\epsilon \cdot 2 \cdot \pi \cdot a}$$

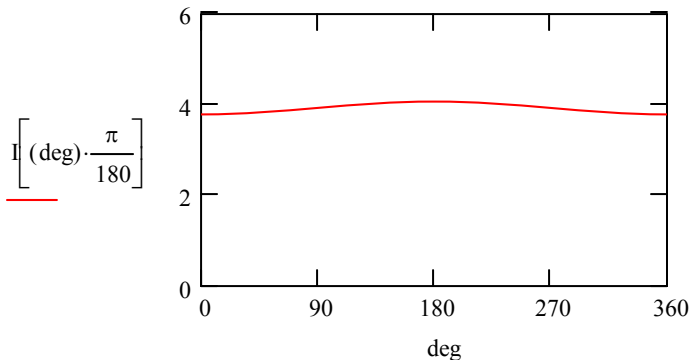
$$H_s(\phi) := J(\phi)$$

$$C_\lambda := k \cdot b \quad I(\phi) := I_o \cdot [1 - 2 \cdot \cos(\phi) \cdot (C_\lambda)^2]$$

$$E_s\left(\frac{\pi}{2}\right) = 1.553i \times 10^4 \quad H_s\left(\frac{\pi}{2}\right) = 153.818$$

deg := 0 .. 360 Loop current around loop circumference

$$I(0) = 3.787$$



$$2 \cdot (k \cdot b)^2 = 0.036$$

loop area, λ^2

$$k \cdot b = 0.134$$

$$I(0) = 3.787$$

$$b = 0.453$$

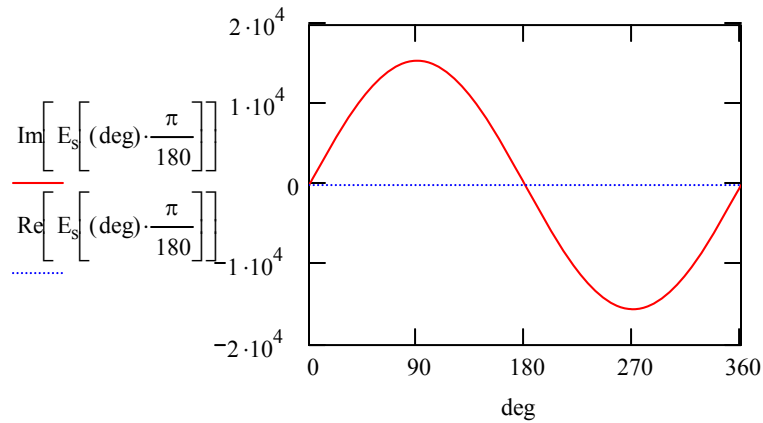
$$I(\pi) = 4.069$$

$$f = 14.1$$

$$I_o = 3.928$$

$$2 \cdot C_\lambda^2 = 0.036$$

deg := 0 .. 360 Surface Electric field around loop circumference, normal to the loop wire:



loop area, λ^2 :

$$2 \cdot (k \cdot b)^2 = 0.036$$

$$k \cdot b = 0.134$$

$$b = 0.453$$

$$f = 14.1$$

$$u := 0.78$$

Peak surface electric field is at:

$$\text{PHI} := \text{root}\left(\frac{d}{du} \text{Im}(E_s(u)), u\right) \quad \text{PHI} \cdot \frac{180}{\pi} = 90 \quad \text{deg.} \quad \text{Esp} := E_s(\text{PHI}) \quad \text{Esp} = 15528.122i$$

Vector potential for the loop in the xy plane:

$$C := \frac{b \cdot \mu}{4 \cdot \pi}$$

$$A_x(x, y, z) := C \cdot \int_0^{2 \cdot \pi} I(\phi) \cdot \frac{e^{-j \cdot k \cdot \sqrt{(x-b \cdot \cos(\phi))^2 + (y-b \cdot \sin(\phi))^2 + z^2}}}{\sqrt{(x-b \cdot \cos(\phi))^2 + (y-b \cdot \sin(\phi))^2 + z^2}} \cdot (-\sin(\phi)) d\phi$$

$$A_y(x, y, z) := C \cdot \int_0^{2 \cdot \pi} I(\phi) \cdot \frac{e^{-j \cdot k \cdot \sqrt{(x-b \cdot \cos(\phi))^2 + (y-b \cdot \sin(\phi))^2 + z^2}}}{\sqrt{(x-b \cdot \cos(\phi))^2 + (y-b \cdot \sin(\phi))^2 + z^2}} \cdot \cos(\phi) d\phi$$

Evaluate the y component of $\nabla \nabla \bullet \mathbf{A}$

$$yA(x, y, z, \phi) := -\sin(\phi) \cdot \left[\begin{aligned} & \frac{3}{4} \cdot i \cdot \frac{k \cdot (2 \cdot x - 2 \cdot b \cdot \cos(\phi)) \cdot (2 \cdot y - 2 \cdot b \cdot \sin(\phi))}{\left[(x - b \cdot \cos(\phi))^2 + (y - b \cdot \sin(\phi))^2 + z^2 \right]^2} \dots \\ & + \frac{-1}{4} \cdot \frac{k^2 \cdot (2 \cdot y - 2 \cdot b \cdot \sin(\phi)) \cdot (2 \cdot x - 2 \cdot b \cdot \cos(\phi))}{\left[(x - b \cdot \cos(\phi))^2 + (y - b \cdot \sin(\phi))^2 + z^2 \right]^2} \dots \\ & + \frac{3}{4} \cdot \frac{(2 \cdot y - 2 \cdot b \cdot \sin(\phi)) \cdot (2 \cdot x - 2 \cdot b \cdot \cos(\phi))}{\left[(x - b \cdot \cos(\phi))^2 + (y - b \cdot \sin(\phi))^2 + z^2 \right]^2} \dots \end{aligned} \right] \left(\frac{3}{2} \right) \\ + \cos(\phi) \cdot \left[\begin{aligned} & \frac{3}{4} \cdot i \cdot \frac{k \cdot (2 \cdot y - 2 \cdot b \cdot \sin(\phi))^2}{\left[(x - b \cdot \cos(\phi))^2 + (y - b \cdot \sin(\phi))^2 + z^2 \right]^2} \dots \\ & + -i \cdot \frac{k}{\left[(x - b \cdot \cos(\phi))^2 + (y - b \cdot \sin(\phi))^2 + z^2 \right]^2} \dots \\ & + \frac{-1}{4} \cdot \frac{k^2 \cdot (2 \cdot y - 2 \cdot b \cdot \sin(\phi))^2}{\left[(x - b \cdot \cos(\phi))^2 + (y - b \cdot \sin(\phi))^2 + z^2 \right]^2} \dots \\ & + \frac{3}{4} \cdot \frac{(2 \cdot y - 2 \cdot b \cdot \sin(\phi))^2}{\left[(x - b \cdot \cos(\phi))^2 + (y - b \cdot \sin(\phi))^2 + z^2 \right]^2} \dots \\ & + \frac{-1}{\left[(x - b \cdot \cos(\phi))^2 + (y - b \cdot \sin(\phi))^2 + z^2 \right]^2} \dots \end{aligned} \right] \left(\frac{5}{2} \right) \left(\frac{3}{2} \right)$$

$$Ey(x, y, z) := -j \cdot \omega \cdot Ay(x, y, z) + \frac{C}{j \cdot \omega \cdot \mu \cdot \epsilon} \int_0^{2 \cdot \pi} I(\phi) \cdot yA(x, y, z, \phi) \cdot e^{-i \cdot k \cdot \sqrt{(x - b \cdot \cos(\phi))^2 + (y - b \cdot \sin(\phi))^2 + z^2}} d\phi$$

Evaluate the x component of $\nabla \nabla \bullet \mathbf{A}$

$$\begin{aligned}
 xA(x, y, z, \phi) := & -\sin(\phi) \cdot \left[\frac{3}{4} \cdot i \cdot \frac{k \cdot (2 \cdot x - 2 \cdot b \cdot \cos(\phi))^2}{\left[(x - b \cdot \cos(\phi))^2 + (y - b \cdot \sin(\phi))^2 + z^2 \right]^2} \dots \right. \\
 & + i \cdot \frac{-k}{\left[(x - b \cdot \cos(\phi))^2 + (y - b \cdot \sin(\phi))^2 + z^2 \right]} \dots \\
 & + \frac{-\frac{1}{4} \cdot k^2 \cdot (2 \cdot x - 2 \cdot b \cdot \cos(\phi))^2}{\left[(x - b \cdot \cos(\phi))^2 + (y - b \cdot \sin(\phi))^2 + z^2 \right]^{\left(\frac{3}{2}\right)}} \dots \\
 & + \frac{3}{4} \cdot \frac{(2 \cdot x - 2 \cdot b \cdot \cos(\phi))^2}{\left[(x - b \cdot \cos(\phi))^2 + (y - b \cdot \sin(\phi))^2 + z^2 \right]^{\left(\frac{5}{2}\right)}} \dots \\
 & + \frac{-1}{\left[(x - b \cdot \cos(\phi))^2 + (y - b \cdot \sin(\phi))^2 + z^2 \right]^{\left(\frac{3}{2}\right)}} \dots \\
 & + \cos(\phi) \cdot \left[\frac{3}{4} \cdot i \cdot \frac{k \cdot (2 \cdot y - 2 \cdot b \cdot \sin(\phi)) \cdot (2 \cdot x - 2 \cdot b \cdot \cos(\phi))}{\left[(x - b \cdot \cos(\phi))^2 + (y - b \cdot \sin(\phi))^2 + z^2 \right]^2} \dots \right. \\
 & + \frac{-\frac{1}{4} \cdot k^2 \cdot (2 \cdot y - 2 \cdot b \cdot \sin(\phi)) \cdot (2 \cdot x - 2 \cdot b \cdot \cos(\phi))}{\left[(x - b \cdot \cos(\phi))^2 + (y - b \cdot \sin(\phi))^2 + z^2 \right]^{\left(\frac{3}{2}\right)}} \dots \\
 & + \frac{3}{4} \cdot \frac{(2 \cdot x - 2 \cdot b \cdot \cos(\phi)) \cdot (2 \cdot y - 2 \cdot b \cdot \sin(\phi))}{\left[(x - b \cdot \cos(\phi))^2 + (y - b \cdot \sin(\phi))^2 + z^2 \right]^{\left(\frac{5}{2}\right)}} \dots \\
 & \left. \left[(x - b \cdot \cos(\phi))^2 + (y - b \cdot \sin(\phi))^2 + z^2 \right]^{\left(\frac{5}{2}\right)} \right]
 \end{aligned}$$

$$E_x(x, y, z) := -j \cdot \omega \cdot A_x(x, y, z) + \frac{C}{j \cdot \omega \cdot \mu \cdot \epsilon} \int_0^{2 \cdot \pi} I(\phi) \cdot xA(x, y, z, \phi) \cdot e^{-i \cdot k \cdot \sqrt{(x - b \cdot \cos(\phi))^2 + (y - b \cdot \sin(\phi))^2 + z^2}} d\phi$$

$$H_y(x, y, z) := \frac{C}{\mu} \cdot \left[\int_0^{2\pi} I(\phi) \cdot (-\sin(\phi)) \cdot \left[\frac{-j \cdot k \cdot z \cdot e^{-i \cdot k \cdot \sqrt{(x-b \cdot \cos(\phi))^2 + (y-b \cdot \sin(\phi))^2 + z^2}}}{(x-b \cdot \cos(\phi))^2 + (y-b \cdot \sin(\phi))^2 + z^2} \dots \right. \right. \\ \left. \left. + \frac{-e^{-i \cdot k \cdot \sqrt{(x-b \cdot \cos(\phi))^2 + (y-b \cdot \sin(\phi))^2 + z^2}}}{\left[(x-b \cdot \cos(\phi))^2 + (y-b \cdot \sin(\phi))^2 + z^2 \right]^{\left(\frac{3}{2}\right)}} \cdot z \right] d\phi \right]$$

$$H_x(x, y, z) := \frac{C}{\mu} \cdot \left[\int_0^{2\pi} I(\phi) \cdot (\cos(\phi)) \cdot \left[\frac{-j \cdot k \cdot z \cdot e^{-i \cdot k \cdot \sqrt{(x-b \cdot \cos(\phi))^2 + (y-b \cdot \sin(\phi))^2 + z^2}}}{(x-b \cdot \cos(\phi))^2 + (y-b \cdot \sin(\phi))^2 + z^2} \dots \right. \right. \\ \left. \left. + \frac{-e^{-i \cdot k \cdot \sqrt{(x-b \cdot \cos(\phi))^2 + (y-b \cdot \sin(\phi))^2 + z^2}}}{\left[(x-b \cdot \cos(\phi))^2 + (y-b \cdot \sin(\phi))^2 + z^2 \right]^{\left(\frac{3}{2}\right)}} \cdot z \right] d\phi \right]$$

$$H_z(x, y, z) := \frac{C}{\mu} \cdot \left[\int_0^{2\pi} I(\phi) \cdot \frac{\sin(\phi)}{2} \cdot \left[\frac{-i \cdot k \cdot (2 \cdot y - 2 \cdot b \cdot \sin(\phi)) \cdot e^{-i \cdot k \cdot \sqrt{(x-b \cdot \cos(\phi))^2 + (y-b \cdot \sin(\phi))^2 + z^2}}}{(x-b \cdot \cos(\phi))^2 + (y-b \cdot \sin(\phi))^2 + z^2} \dots \right. \right. \\ \left. \left. + \frac{-(2 \cdot y - 2 \cdot b \cdot \sin(\phi)) \cdot e^{-i \cdot k \cdot \sqrt{(x-b \cdot \cos(\phi))^2 + (y-b \cdot \sin(\phi))^2 + z^2}}}{\left[(x-b \cdot \cos(\phi))^2 + (y-b \cdot \sin(\phi))^2 + z^2 \right]^{\left(\frac{3}{2}\right)}} \right] d\phi \dots \right. \\ \left. + \int_0^{2\pi} I(\phi) \cdot \frac{\cos(\phi)}{2} \cdot \left[\frac{-i \cdot k \cdot (2 \cdot x - 2 \cdot b \cdot \cos(\phi)) \cdot e^{-i \cdot k \cdot \sqrt{(x-b \cdot \cos(\phi))^2 + (y-b \cdot \sin(\phi))^2 + z^2}}}{(x-b \cdot \cos(\phi))^2 + (y-b \cdot \sin(\phi))^2 + z^2} \dots \right. \right. \\ \left. \left. + \frac{-(2 \cdot x - 2 \cdot b \cdot \cos(\phi)) \cdot e^{-i \cdot k \cdot \sqrt{(x-b \cdot \cos(\phi))^2 + (y-b \cdot \sin(\phi))^2 + z^2}}}{\left[(x-b \cdot \cos(\phi))^2 + (y-b \cdot \sin(\phi))^2 + z^2 \right]^{\left(\frac{3}{2}\right)}} \right] d\phi \right]$$

$$ET(x, y, z) := \sqrt{(|Ex(x, y, z)|)^2 + (|Ey(x, y, z)|)^2} \quad E_s(\phi) := \frac{q(\phi)}{\varepsilon \cdot 2 \cdot \pi \cdot a}$$

$$HT(x, y, z) := \sqrt{(|Hx(x, y, z)|)^2 + (|Hy(x, y, z)|)^2 + (|Hz(x, y, z)|)^2} \quad H_s(\phi) := J(\phi) \quad u := 10$$

Check if the field point is within u% of inside of the loop toroidal surface, replace with the value at the surface for later computational convenience:

$$\text{inToroid}(x, y, z) := \text{if} \left[\sqrt{(\sqrt{x^2 + y^2} - b)^2 + z^2} \leq a \cdot \left(1 + \frac{u}{100} \right), 1, 0 \right]$$

$$Et(x, y, z) := \text{if} \left[\text{inToroid}(x, y, z), |E_s(\arg(x + j \cdot y))|, \sqrt{(|Ex(x, y, z)|)^2 + (|Ey(x, y, z)|)^2} \right]$$

$$Ht(x, y, z) := \text{if} \left[\text{inToroid}(x, y, z), |H_s(\arg(x + j \cdot y))|, \sqrt{(|Hx(x, y, z)|)^2 + (|Hy(x, y, z)|)^2 + (|Hz(x, y, z)|)^2} \right]$$

Sanity Checks... at large distance, dd: $dd := \frac{1000}{k}$ $k = 0.296$ $f = 14.1$

For vanishingly small loop: $E_{\text{phi}}(r) := \eta \cdot \frac{k^3 \cdot I_0 \cdot \pi \cdot b^2}{4 \cdot \pi} \cdot \left[\frac{1}{k \cdot r} - \frac{j}{(k \cdot r)^2} \right]$ $\frac{|E_{\text{phi}}(dd)|}{Et(0, dd, 0)} = 1.002$

Solenoid: $H_{000} := \frac{I_0}{2 \cdot b}$ $H_{000} = 4.33151$ $E_{000} := -j \cdot \eta \cdot \frac{k}{2} \cdot I_0$ $E_{000} = -218.63545i$

Exact: $H_z(0, 0, 0) = 4.370219 - 3.466485i \times 10^{-3}$ $E_y(0, 0, 0) = 0.349 - 216.699i$
 $E_x(0, 0, 0) = 1.04323 \times 10^{-13} + 8.86085i \times 10^{-13}$

Wave impedance at center of loop: $\frac{Et(0, 0, 0)}{Ht(0, 0, 0)} = 49.58553$ $Z_{000} := -j \cdot k \cdot b \cdot \eta$ $|Z_{000}| = 50.4755177$

$$Z(k, b, a) + Z_{\text{loss}} = 0.217329 + 256.69636i$$

resonating cap voltage:

$$QL := 0.5 \cdot \frac{\text{Im}(Z(k, b, a))}{\text{Re}(Z(k, b, a)) + R_{\text{loss}}} \quad QL = 590.242$$

$$V_{\text{cap}} := \sqrt{2 \cdot \text{Im}(Z(k, b, a)) \cdot QL \cdot P_w}$$

$$V_{\text{cap}} = 1230.5639 \quad V \text{ peak, for } P_w \text{ not } P!$$

Far field distance, m: $dd := \lambda \cdot 10^9$

$$P = 5 \quad P_w = 5$$

Far Field check: $\frac{Ht(dd, dd, 0)}{\sqrt{\frac{P \cdot \text{eff} \cdot 1.5}{4 \cdot \pi \cdot \eta}}} = 0.93738$
ratio approaches 1
for extremely small
loop: $\frac{dd \cdot \sqrt{2}}{dd \cdot \sqrt{2}}$

$$\frac{Et(dd, dd, 0)}{\sqrt{\frac{P \cdot \text{eff} \cdot 1.5 \cdot \eta}{4 \cdot \pi}}} = 0.93738$$

Approximate far field null depth: $\text{nullFAR} := 20 \cdot \log[2(k \cdot b)]$ $\text{nullFAR} = -11.438$

$$\text{null4nec2}_{14} := -11.5 \quad \text{dB at 14.1 MHz}$$

"Null depth" in the far field (loop is in xy-plane), on axis, relative to peak in x, peak in z:

$$dd = 2.126 \times 10^{10} \quad \text{m} \quad (k \cdot b) = 0.134 \quad f = 14.1$$

$$\frac{Ht(0,0,dd)}{Ht(dd,0,0)} = 0.259495 \quad \text{Null} := 20 \cdot \log\left(\frac{Ht(0,0,dd)}{Ht(dd,0,0)}\right) \quad \text{Null} = -11.717 \quad \text{dB}$$

$$\frac{Ht(0,0,dd)}{Ht(0,dd,0)} = 0.268568 \quad \text{Null} := 20 \cdot \log\left(\frac{Ht(0,0,dd)}{Ht(0,dd,0)}\right) \quad \text{Null} = -11.419 \quad \text{dB}$$

$$\text{nullFAR} = -11.438$$

$$\frac{Et(0,0,dd)}{Et(0,dd,0)} = 0.268568 \quad \text{Null} := 20 \cdot \log\left(\frac{Et(0,0,dd)}{Et(0,dd,0)}\right) \quad \text{Null} = -11.419 \quad \text{dB}$$

$$\frac{Et(0,0,dd)}{Et(dd,0,0)} = 0.259495 \quad \text{Null} := 20 \cdot \log\left(\frac{Et(0,0,dd)}{Et(dd,0,0)}\right) \quad \text{Null} = -11.717 \quad \text{dB}$$

Perturb the far field location by dx around x and y, seek for off axis null:

$$\text{NullX}(dx) := 20 \cdot \log\left(\frac{Ht(dx,0,dd)}{Ht(0,dd,0)}\right) \quad \text{NullX}(0) = -11.419 \quad \text{dB}$$

$$\text{NullY}(dx) := 20 \cdot \log\left(\frac{Ht(0,dx,dd)}{Ht(0,dd,0)}\right) \quad \text{NullY}(0) = -11.419 \quad \text{dB}$$

Off the main axes:

$$\text{NullOffp}(dx) := 20 \cdot \log\left(\frac{Ht\left(\frac{dx}{\sqrt{2}}, \frac{dx}{\sqrt{2}}, dd\right)}{Ht(0,dd,0)}\right) \quad \text{NullOffp}(0) = -11.419 \quad \text{dB}$$

$$\text{NullOffm}(dx) := 20 \cdot \log\left(\frac{Ht\left(\frac{-dx}{\sqrt{2}}, \frac{dx}{\sqrt{2}}, dd\right)}{Ht(0,dd,0)}\right) \quad \text{NullOffm}(0) = -11.419 \quad \text{dB}$$

$$dxi := 0..10 \quad \text{scale} := 100 \quad DX_{dxi} := (-5 + dxi) \cdot \text{scale}$$

$$DX_{dxi} = \text{NullX}(DX_{dxi}) = \text{NullY}(DX_{dxi}) = \text{NullOffp}(DX_{dxi}) = \text{NullOffm}(DX_{dxi}) = \quad f = 14.1$$

-500	-11.419	-11.419	-11.419	-11.419
-400	-11.419	-11.419	-11.419	-11.419
-300	-11.419	-11.419	-11.419	-11.419
-200	-11.419	-11.419	-11.419	-11.419
-100	-11.419	-11.419	-11.419	-11.419
0	-11.419	-11.419	-11.419	-11.419
100	-11.419	-11.419	-11.419	-11.419
200	-11.419	-11.419	-11.419	-11.419
300	-11.419	-11.419	-11.419	-11.419
400	-11.419	-11.419	-11.419	-11.419
500	-11.419	-11.419	-11.419	-11.419

**Search result:
the null is
symmetrical, and on
the x=0, y=0, z-axis**

**Omitting the ϕ
current term would
results in an
extremely deep null!**

Near Fields: $dx_{dxi} := -0.7 + dxi \cdot 0.14$ $b = 0.453$

The loop gap is in the +x-axis

There is symmetry about y-axis

$dx_{dxi} =$ $Et(dx_{dxi}, 0, 0) =$ $Ht(dx_{dxi}, 0, 0) = \frac{Et(dx_{dxi}, 0, 0)}{Ht(dx_{dxi}, 0, 0)} =$ $Et(0, dx_{dxi}, 0) =$ $Ht(0, dx_{dxi}, 0) = \frac{Et(0, dx_{dxi}, 0)}{Ht(0, dx_{dxi}, 0)} =$

-0.7	13.434	1.125	11.94	191.937	1.065	180.251
-0.56	59.715	3.865	15.45	556.084	3.7	150.3
-0.42	214.779	22.871	9.391	$1.825 \cdot 10^3$	22.136	82.461
-0.28	176.182	6.451	27.312	380.002	6.312	60.207
-0.14	189.411	4.761	39.781	245.267	4.71	52.079
0	216.7	4.37	49.586	216.7	4.37	49.586
0.14	260.087	4.658	55.84	245.267	4.71	52.079
0.28	338.108	6.172	54.777	380.002	6.312	60.207
0.42	600.229	21.401	28.047	$1.825 \cdot 10^3$	22.136	82.461
0.56	269.408	3.535	76.222	556.084	3.7	150.3
0.7	126.935	1.005	126.364	191.937	1.065	180.251

Loop center: i=5

$dx_5 = 0$ $Et(dx_5, 0, 0) = 216.7$ $Ht(dx_5, 0, 0) = 4.37$

$Et(0, dx_5, 0) = 216.7$ $Ht(0, dx_5, 0) = 4.37$

$Z000 := -j \cdot k \cdot b \cdot \eta$ $|Z000| = 50.4755177$ $\frac{Et(dx_5, 0, 0)}{Ht(dx_5, 0, 0)} = 49.586$

$\frac{Et(0, dx_5, 0)}{Ht(0, dx_5, 0)} = 49.586$